

MATH 5061 Problem Set 1¹

Due date: Feb 5, 2024

Problems: (Please hand in your assignments by submitting your PDF via email. **Late submissions will not be accepted.**)

1. Show that \mathbb{S}^2 and $\mathbb{C}\mathbb{P}^1$ are diffeomorphic by constructing an explicit diffeomorphism between them.
2. Prove that the tangent bundle TM is always orientable as a manifold.
3. Prove *Jacobi identity*: $[X, [Y, Z]] + [Y, [Z, X]] + [Z, [X, Y]] = 0$ for any $X, Y, Z \in \Gamma(TM)$.
4. Let α be a $(0, q)$ -tensor on M , $X, Y_1, \dots, Y_q \in \Gamma(TM)$ be vector fields. Show that

$$(\mathcal{L}_X \alpha)(Y_1, \dots, Y_q) = X(\alpha(Y_1, \dots, Y_q)) - \sum_{i=1}^q \alpha(Y_1, \dots, Y_{i-1}, [X, Y_i], Y_{i+1}, \dots, Y_q).$$

¹Last revised on January 21, 2024